An improved delay and state observer for SISO LTI systems with known delay lower bound

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Abstract

It is known that the presence of delays hinders the performance achievable by a feedback control system, and it can even lead to closed-loop instability if not considered during the design. For this reason, predictors are often included in the loop, although they typically require the knowledge of the exact value of the delay, which in some applications is hard to obtain in practice. This paper presents a method to design an observer that simultaneously estimates the unknown state and the time-varying input delay of a plant based on an available model and the measurements coming from the sensors. In particular, the main contribution of this paper is to show that by accounting for a known lower bound of the input delay, it is possible to improve the observer's performance when compared to state-of-the-art approaches encountered in the literature. Simulations are used to illustrate the efficiency of the proposed design method.
Keywords: input delay, delay estimation, state observer

1 Introduction

An input delay system is a particular case of time-delay system in which the delay affects only the input signal. There are many possible causes for input delays, of both computational and physical nature. In many situations, these delays must be explicitly considered during the design, because failing to do so can lead to a non-acceptable degradation of the performance or to the loss of stability.

The most widely used approach to account for delays is the Smith predictor (Smith, 1959) which was extended to work with time-varying delays in (Normey-Rico et al., 2012) and (Franklin and Santos, 2020). An alternative approach that has drawn attention lately relies on the linear parameter varying (LPV) framework (Rotondo et al., 2019) and achieves the desired performance/stabilization using a delay-scheduled controller (Briat et al., 2009b,a). However, even when the time delay is explicitly taken into account, the presence of uncertainty on the delay value can produce fragility in the control system. A possible way to address this issue and decrease the amount of uncertainty in the delay value is to estimate it online. For this reason, many works in the literature have focused on the problem of time delay estimation.

The approaches used for time delay estimation can be classified into signal processing and control-oriented (Léchappé et al., 2018). In the former class of methods, the value of the delay that minimizes some criterion (cost function) based on available data collected from the process is sought (Knapp and Carter, 1976; Jacovitti and Scarano, 1993). However, this comes at the cost of a high computation time and they require the knowledge of the delayed signal, which is not always realistic. For this reason, control-oriented approaches consider the delay as a parameter to be estimated online. (Agarwal and Canudas, 1987) approximated the delay term using the Padé form, then used least-squares to minimize an objective function. (Tuch et al., 1994) proposed a recursive least-squares algorithm in the frequency domain, which had the drawback of requiring perfectly known initial conditions.

Among the control-oriented approaches, observer-based methods have been under consideration lately. For instance, (Cacace et al., 2015) proposed an augmented observer able to estimate a constant state delay. A Kalman filter-based solution was proposed in (Léchappé et al., 2015) and (Léchappé et al., 2018). On the other hand, adaptive and sliding mode observers were applied to time delay estimation in (Wu et al., 2013) and (Drakunov et al., 2006).

In this paper, we consider the observer-based solution proposed by (Léchappé et al., 2015), and we improve it under the assumption that a known lower bound is available for the time-varying delay. We discuss how this knowledge can be used when applying the Taylor's theorem, so that a generally smaller remainder is obtained. Consequently, a better performing observer is implemented, as demonstrated by simulations under different realizations of the delay and input signals.

The paper is organized as follows. Section 2 presents the problem formulation and summarizes the state and delay observer proposed by (Léchappé et al., 2015). Section 3 shows that by considering a lower bound for the delay, it is possible to perform a different sequence of calculations that lead to an improved state and delay observer structure. The discussion about the observer stability is provided in Section 4. The performance of the proposed improved observer is demonstrated using simulation examples in Section 5. Finally, the main conclusions are drawn in Section 6.

2 Problem and background

Consider the following SISO LTI system:

$$\begin{cases} \dot{x}(t) = Ax(t) + bu(t - d(t)) \\ y(t) = c^T x(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the known input, $y \in \mathbb{R}$ is the measured output, and $d \in \mathbb{R}$ is the unknown input delay, which should be estimated. The matrix A and the vectors b, c are assumed to be known and such that the pair (A, c^T) is observable. On the other hand, it is assumed that the time-varying delay $d(t)$ is continuous and differentiable, with $|\dot{d}(t)| \leq H$. Finally, it is assumed that the signal $u(t)$ is smooth, which means $u \in \mathscr{C}^2$ and there exists a bound $M > 0$ such that $|\ddot{u}(t)| \leq M$ for all $t \geq -\overline{d}$.

The problem under consideration is the design of an observer that can estimate the state $x(t)$ and the input delay $d(t)$ from the knowledge of $y(t)$, $u(t)$ and $\dot{u}(t)$.

The work by (Léchappé et al., 2015) assumed that the time-varying delay $d(t)$ satisfies $d(t) \in [0, \overline{d}]$, where $\overline{d} > 0$ denotes the upper bound of $d(t)$. Then, they proceed to apply the Taylor's theorem to expand $u(t - d(t))$ about t, obtaining:

$$u(t - d(t)) = u(t) - d(t)\dot{u}(t) + \gamma(t - d(t)) \tag{2}$$

where $\gamma(\cdot)$ is the so-called *remainder*, for which a uniform bound can be obtained given that $|\ddot{u}(t)| \leq M$, as follows:

$$|\gamma(t - d(t))| \leq M\frac{d(t)^2}{2} \tag{3}$$

This allowed (Léchappé et al., 2015) to rewrite (1) as:

$$\begin{cases} \dot{x}(t) = Ax(t) + bu(t) - b\dot{u}(t)d(t) + b\gamma(t - d(t)) \\ y(t) = c^T x(t) \end{cases} \tag{4}$$

and, by augmenting the state vector as $z(t) = \begin{bmatrix} x(t)^T & d(t)^T \end{bmatrix}^T$:

$$\begin{cases} \dot{z}(t) = \begin{bmatrix} A & -b\dot{u}(t) \\ 0 & 0 \end{bmatrix} z(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} b\gamma(t - d(t)) \\ \dot{d}(t) \end{bmatrix} \\ y(t) = \begin{bmatrix} c^T & 0 \end{bmatrix} z(t) \end{cases} \tag{5}$$

which can be brought to the compact form:

$$\begin{cases} \dot{z}(t) = \bar{A}(\dot{u}(t))z(t) + \bar{B}u(t) + \Gamma(t, t - d(t)) \\ y(t) = \bar{C}z(t) \end{cases} \tag{6}$$

where the definition of $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\Gamma(\cdot)$ is straightforward.

Then, the following observer was proposed to reconstruct and estimate $\hat{z}(t)$ of $z(t)$ (and, in turn, to obtain estimates $\hat{x}(t)$ and $\hat{d}(t)$ of $x(t)$ and $d(t)$):

$$\dot{\hat{z}}(t) = \bar{A}(\dot{u}(t))\hat{z}(t) + \bar{B}u(t) - L(t)\left(\bar{C}\hat{z}(t) - y(t)\right) \tag{7}$$

where the discussion about how to choose the observer gain L will be omitted, since it is not relevant for further developments (the interested reader is referred to (Léchappé et al., 2015)).

3 Improved state and delay observer

As already mentioned, the approach proposed by (Léchappé et al., 2015) and summarized in the previous section works under the assumption that $d(t) \in [0, \overline{d}]$. In many practical situations, it is realistic to assume that not only an upper bound $\overline{d}$ is available, but a lower bound $\underline{d}$ as well, which means that $d(t) \in [\underline{d}, \overline{d}]$. In this case, expanding $u(t - d(t))$ about $t - \underline{d}$ would lead to a reduced magnitude of the remainder term, which acts as an unknown exogenous disturbance in the model (5), thus being responsible for degrading the state/delay estimate obtained via the observer (7).

Then, the following is obtained:

$$\begin{aligned} u(t - d(t)) =& u(t - \underline{d}) - d(t)\dot{u}(t - \underline{d}) \\ &+ \underline{d}\dot{u}(t - \underline{d}) + \tilde{\gamma}(t - d(t)) \end{aligned} \tag{8}$$

where $\tilde{\gamma}(\cdot)$ is a new remainder term, for which a uniform bound is calculated as follows:

$$|\tilde{\gamma}(t - d(t))| \leq M\frac{(d(t) - \underline{d})^2}{2} \tag{9}$$

By comparing (3) and (9), it is clear that given the same value of the delay $d(t)$, it can be generally expected that:

$$|\tilde{\gamma}(t - d(t))| \leq |\gamma(t - d(t))| \tag{10}$$

will likely hold.

Based on (8), Eq. (1) can be rewritten as:

$$\begin{cases} \dot{x}(t) = Ax(t) + bu(t - \underline{d}) - b\dot{u}(t - \underline{d})d(t) \\ \qquad + b\underline{d}\dot{u}(t - \underline{d}) + b\gamma(t - d(t)) \\ y(t) = c^T x(t) \end{cases} \tag{11}$$

and, using the augmented state vector $z(t)$:

$$\begin{cases} \dot{z}(t) = \begin{bmatrix} A & -b\dot{u}(t - \underline{d}) \\ 0 & 0 \end{bmatrix} z(t) \\ \qquad + \begin{bmatrix} b & b\underline{d} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u(t - \underline{d}) \\ \dot{u}(t - \underline{d}) \end{bmatrix} + \begin{bmatrix} b\tilde{\gamma}(t - d(t)) \\ \dot{d}(t) \end{bmatrix} \\ y(t) = \begin{bmatrix} c^T & 0 \end{bmatrix} z(t) \end{cases} \tag{12}$$

which can be brought to a form similar to (6):

$$\begin{cases} \dot{z}(t) = \bar{A}(\dot{u}(t - \underline{d}))z(t) + \bar{B}\begin{bmatrix} u(t - \underline{d}) \\ \dot{u}(t - \underline{d}) \end{bmatrix} + \tilde{\Gamma}(t, t - d(t)) \\ y(t) = \bar{C}z(t) \end{cases} \tag{13}$$

with an appropriate definition of the matrices $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\tilde{\Gamma}(\cdot)$.

For the system (13), let us use the following observer:

$$\dot{\hat{z}}(t) = \bar{A}(\dot{u}(t - \underline{d}))\hat{z}(t) + \bar{B}\begin{bmatrix} u(t - \underline{d}) \\ \dot{u}(t - \underline{d}) \end{bmatrix} - L(t)\left(\bar{C}\hat{z}(t) - y(t)\right) \tag{14}$$

where, following (Léchappé et al., 2015), the gain $L(t)$ is chosen according to a Kalman filter-like structure, i.e.:

$$L(t) = S(t)^{-1}\bar{C}^T R \tag{15}$$

where R is a positive diagonal matrix that acts as a filter, chosen as $R = I$ in the noise-free scenario, and the symmetric matrix S is obtained as the solution of the following matrix differential equation:

$$\begin{aligned}\dot{S}(t) = &-\rho S(t) - \bar{A}\left(\dot{u}(t-\underline{d})\right)^T S(t) \\ &- S(t)\bar{A}\left(\dot{u}(t-\underline{d})\right) + \bar{C}^T R\bar{C}\end{aligned} \tag{16}$$

with a positive constant $\rho > 0$ that affects the convergence speed of S and an initial condition $S(0) \succ 0$.

4 Observer stability

This section provides the observer stability proof, which follows the theoretical steps discussed in (Léchappé et al., 2015). For the sake of proving the stability, let us recall the following lemma (Khalil, 2002).

Lemma 1. *Let $x = 0$ be an exponentially stable equilibrium point of the nominal system:*

$$\dot{x}(t) = f\left(t, x(t)\right) \tag{17}$$

and let $V\left(t, x(t)\right)$ be a Lyapunov function for (17) that satisfies for all $t \geq 0$ and $\forall x \in \mathscr{D} = \{x \in \mathbb{R}^n : \|x\|_2 < r\}$:

$$c_1 \|x\|^2 \leq V\left(t, x(t)\right) \leq c_2 \|x\|^2 \tag{18}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f\left(t, x(t)\right) \leq -c_3 \|x\|^2 \tag{19}$$

$$\left\| \frac{\partial V}{\partial x} \right\| \leq c_4 \|x\| \tag{20}$$

for some positive constants c_1, c_2, c_3, c_4. Also, assume that a perturbation term $g(t,x)$ satisfies:

$$\|g(t,x)\| \leq \gamma(t)\|x\| + \delta(t) \quad \forall t \geq 0, \forall x \in \mathscr{D} \tag{21}$$

where $\gamma : \mathbb{R} \rightarrow \mathbb{R}$ is a nonnegative and continuous function such that:

$$\int_{t_0}^{t} \gamma(\tau) d\tau \leq \varepsilon(t - t_0) + \eta \tag{22}$$

for some nonnegative constants ε and η where:

$$\varepsilon < \frac{c_1 c_3}{c_2 c_4} \tag{23}$$

and $\delta : \mathbb{R} \rightarrow \mathbb{R}$ is nonnegative, continuous, and bounded for all $t \geq 0$. Provided that:

$$\|x(t_0)\| < \frac{r}{\rho}\sqrt{\frac{c_1}{c_2}} \tag{24}$$

$$\sup_{t \geq t_0} \delta(t) < \frac{2 c_1 \theta r}{c_4 \rho} \tag{25}$$

with:

$$\theta = \frac{1}{2}\left[\frac{c_3}{c_2} - \varepsilon \frac{c_4}{c_1}\right] > 0 \tag{26}$$

$$\rho = \exp\left(\frac{c_4 \eta}{2 c_1}\right) \geq 1 \tag{27}$$

then, the solution of the perturbed system:

$$\dot{x}(t) = f\left(t, x(t)\right) + g\left(t, x(t)\right) \tag{28}$$

satisfies:

$$\|x(t)\| < \sqrt{\frac{c_2}{c_1}}\rho \|x(t_0)\| e^{-\theta(t-t_0)} + \frac{c_4 \rho}{2 c_1}\int_{t_0}^{t} e^{-\theta(t-\tau)}\delta(\tau) d\tau \tag{29}$$

In addition, let us recall the following lemma given by (Besançon et al., 1996), which characterizes the matrix $S(t)$ obtained as a solution of (16).

Lemma 2. *Assume that $\dot{u}$ is regularly persistent, and consider the differential equation (16). Then $\exists \rho_0 > 0$ such that for any symmetric positive definite matrix $S(0)$, $\forall \rho \geq \rho_0$, there exist $\bar{\alpha}, \bar{\beta}, t_0 > 0$ such that $\forall t \geq t_0$:*

$$\bar{\alpha} I \preceq S(t) \preceq \bar{\beta} I \tag{30}$$

Let us define the Lyapunov candidate function:

$$V\left(e(t)\right) = e(t)^T S(t) e(t) \tag{31}$$

where $S(t)$ is the solution of (16) and $e(t) = \hat{z}(t) - z(t)$. Given the assumption of regularly persistent $\dot{u}(t)$, in virtue of Lemma 2, there exists a positive scalar ρ_0 so that $\forall \rho \geq \rho_0$ there exist $\bar{\alpha}, \bar{\beta}, t_0 > 0$ such that $\forall t \geq t_0$:

$$\bar{\alpha}\|e(t)\|^2 \leq V\left(e(t)\right) \leq \bar{\beta}\|e(t)\|^2 \tag{32}$$

so that (18) holds with $c_1 = \bar{\alpha}$ and $c_2 = \bar{\beta}$. The dynamics of the estimation error is described by:

$$\dot{e}(t) = \dot{\hat{z}}(t) - \dot{z}(t) \tag{33}$$

which, using (13)-(15), follows:

$$\begin{aligned}\dot{e}(t) &= \bar{A}\left(\dot{u}(t-\underline{d})\right)\hat{z}(t) + \bar{B}\begin{bmatrix} u(t-\underline{d}) \\ \dot{u}(t-\underline{d}) \end{bmatrix} - L(t)\left(\bar{C}\hat{z}(t) - y(t)\right) \\ &\quad - \bar{A}\left(\dot{u}(t-\underline{d})\right) z(t) - \bar{B}\begin{bmatrix} u(t-\underline{d}) \\ \dot{u}(t-\underline{d}) \end{bmatrix} - \tilde{\Gamma}\left(t, t-d(t)\right) \\ &= \left(\bar{A}\left(\dot{u}(t-\underline{d})\right) - L(t)\bar{C}\right) e(t) - \tilde{\Gamma}\left(t, t-d(t)\right) \\ &= \left(\bar{A}\left(\dot{u}(t-\underline{d})\right) - S(t)^{-1}\bar{C}^T R\bar{C}\right) e(t) - \tilde{\Gamma}\left(t, t-d(t)\right)\end{aligned} \tag{34}$$

Let us neglect the perturbation term $\tilde{\Gamma}\left(t, t-d(t)\right)$ so that:

$$\dot{V}\left(e(t)\right) = \dot{e}(t)^T S(t) e(t) + e(t)^T \dot{S}(t) e(t) + e(t)^T S(t)\dot{e}(t) \tag{35}$$

becomes, after taking into account (16) and (34):

$$\begin{aligned}\dot{V}\left(e(t)\right) &= e(t)^T\left[\bar{A}\left(u(t-\underline{d})\right) - S(t)^{-1}\bar{C}^T R\bar{C}\right]^T S(t) e(t) \\ &\quad - \rho e(t)^T S(t) e(t) - e(t)^T \bar{A}\left(\dot{u}(t-\underline{d})\right)^T S(t) e(t) \\ &\quad - e(t)^T S(t)\bar{A}\left(\dot{u}(t-\underline{d})\right) e(t) + e(t)^T \bar{C}^T R\bar{C} e(t) \\ &\quad + e(t)^T S(t)\left[\bar{A}\left(u(t-\underline{d})\right) - S(t)^{-1}\bar{C}^T R\bar{C}\right] e(t) \\ &= -\rho e(t)^T S(t) e(t) - e(t)^T \bar{C}^T R\bar{C} e(t)\end{aligned} \tag{36}$$

Due to the positive definiteness of the matrix $\bar{C}^T R\bar{C}$, (32) and (36) lead to:

$$\dot{V}(e(t)) \leq -\rho\bar{\alpha}\|e(t)\|^2 \tag{37}$$

so that (19) holds with $c_3 = \rho\bar{\alpha}$. Finally, from (32) it follows that:

$$\left\|\frac{\partial V(e(t))}{\partial e}\right\| \leq 2\bar{\beta}\|e(t)\| \tag{38}$$

which means that (20) holds with $c_4 = 2\bar{\beta}$.

Let us consider now the perturbation term $\tilde{\Gamma}(t, t-d(t))$. Notably, in this case the domain $\mathscr{D}$ corresponds to the entire state-space ($r=\infty$), which means that by choosing $\gamma(t) = 0$ and $\delta(t) = \sup\|\tilde{\Gamma}(\cdot)\|$ in (21), then (24)-(25) always hold and we can choose $\varepsilon = \eta = 0$ in (22)-(23). Consequently, (26)-(27) lead to $\rho = 1$ and $\theta = \bar{\alpha}/2\bar{\beta}$. Then, according to Lemma 1, $e(t)$ will satisfy the following inequality:

$$\begin{aligned}\|e(t)\| &< \sqrt{\frac{\bar{\beta}}{\bar{\alpha}}}\|e(t_0)\| e^{-\frac{\bar{\alpha}}{2\bar{\beta}}(t-t_0)} \\ &+ \frac{\bar{\beta}}{\bar{\alpha}}\int_{t_0}^{t} e^{-\frac{\bar{\alpha}}{2\bar{\beta}}(t-\tau)} \sup\|\tilde{\Gamma}(\cdot)\| d\tau \\ &= \sqrt{\frac{\bar{\beta}}{\bar{\alpha}}}\|e(t_0)\| e^{-\frac{\bar{\alpha}}{2\bar{\beta}}(t-t_0)} \\ &+ \frac{\bar{\beta}}{\bar{\alpha}}\int_{0}^{t-t_0} e^{-\frac{\bar{\alpha}}{2\bar{\beta}}s} \sup\|\tilde{\Gamma}(\cdot)\| ds \\ &= \sqrt{\frac{\bar{\beta}}{\bar{\alpha}}}\|e(t_0)\| e^{-\frac{\bar{\alpha}}{2\bar{\beta}}(t-t_0)} \\ &+ \frac{2\bar{\beta}^2}{\bar{\alpha}^2}\left(1 - e^{-\frac{\bar{\alpha}}{2\bar{\beta}}(t-t_0)}\right) \sup\|\tilde{\Gamma}(\cdot)\|\end{aligned} \tag{39}$$

which shows the ultimate boundedness of the estimation error $e(t)$.

It is worth remarking that the regular persistency of $\dot{u}(t)$ is a key point for the above discussion to hold true, which is the reason why in practice the estimation error spikes at times where this assumption does not hold true, as shown in the results in the next section.

5 Simulation results

Let us consider the second-order example proposed by (Léchappé et al., 2015):

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-d(t)) \tag{40}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \tag{41}$$

for which we assume that $x(0) = \begin{bmatrix}1.5, 1\end{bmatrix}^T$. Then, the extended system is described by matrices:

$$\bar{A}(\dot{u}) = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -3 & -\dot{u} \\ 0 & 0 & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 & 0 \\ 1 & \underline{d} \\ 0 & 0 \end{bmatrix} \tag{42}$$

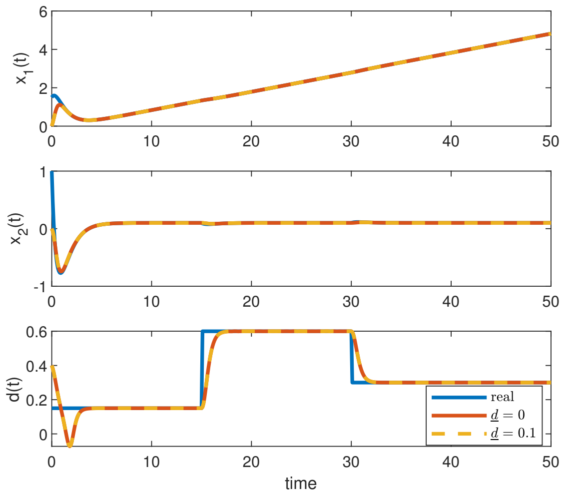

Figure 1. Simulation results (Scenario 1).

where $\underline{d}$ is the known lower bound for $d(t)$. We will assume that the observer (14)-(16) is obtained with $\rho = 5$, $S(0) = I_3$, $\hat{x}(0) = [0,0]^T$, $\hat{d}(0) = 0.4$, and we will compare the performance of the observer proposed in this paper with the results in (Léchappé et al., 2015).

5.1 Scenario 1

Let us first consider the delay sequence:

$$d(t) = \begin{cases} 0.15 & \text{for } 0 \leq t \leq 15 \\ 0.6 & \text{for } 15 < t \leq 30 \\ 0.3 & \text{otherwise} \end{cases} \tag{43}$$

with a ramp input signal $u(t) = 0.2t$ for which $\dot{u}(t) = 0.2$, and let us consider a known lower bound for the delay $\underline{d} = 0.1$. Under these conditions, we obtain the simulation results shown in Figure 1. In this scenario, the relationship (2) holds with a zero remainder term, so that (6) reduces to:

$$\dot{z}(t) = \bar{A}(\dot{u}(t))z(t) + \bar{B}u(t) \tag{44}$$

and the estimation errors for both the state variables and the delay tend asymptotically to zero. Notably, no difference between the case with $\underline{d} = 0$ and $\underline{d} \neq 0$ is perceivable in this case.

5.2 Scenario 2

Let us now consider the delay $d(t) = 0.4 + 0.2\sin(0.4t)$ while keeping the ramp input signal $u(t) = 0.2t$, with the known lower bound for the delay $\underline{d} = 0.2$. In this case, $\dot{d}(t) \neq 0$ acts as an exogenous disturbance that prevents the observer from estimating the delay correctly, as shown in Figure 2, where $d(t) - \hat{d}(t)$ exhibits a clear steady-state error. Notably, also in this case the knowledge of a precise lower bound $\underline{d}$ for $d(t)$ does not play any role.

5.3 Scenario 3

Let us now consider the following modification to Scenario 1: $u(t) = \sin(0.1t)$, so that the terms $\gamma(t-d(t))$

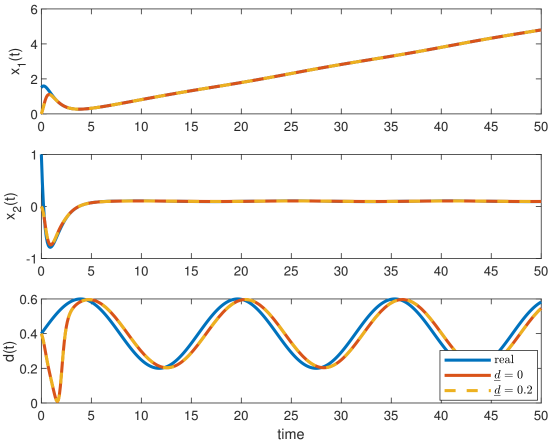

Figure 2. Simulation results (Scenario 2).

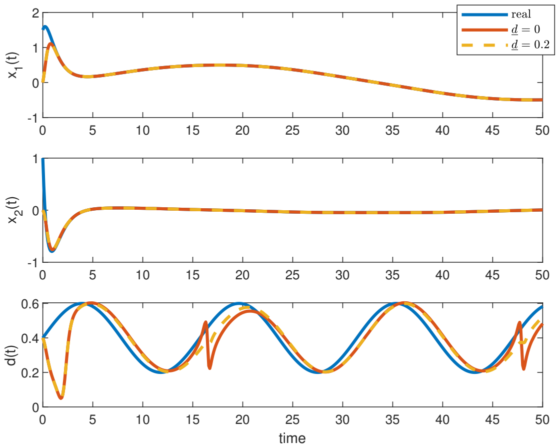

Figure 4. Simulation results (Scenario 4).

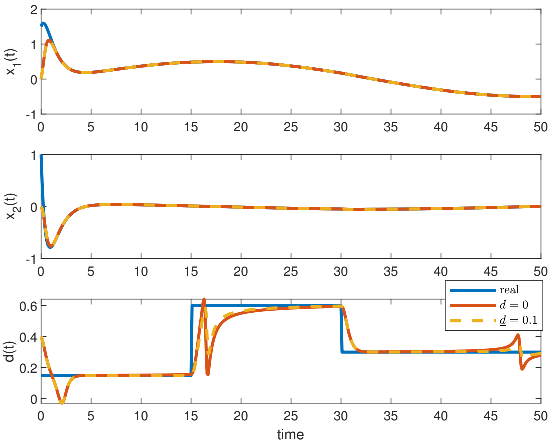

Figure 3. Simulation results (Scenario 3).

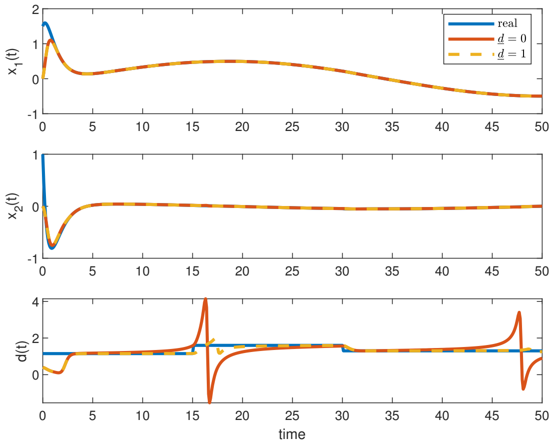

Figure 5. Simulation results (Scenario 5).

and $\tilde{y}(t-d(t))$ appear in (2) and (8), respectively. Given that (10) is likely to happen, we would now expect to see a difference between the performance of the observer with $\underline{d}=0$ and that of the observer with $\underline{d}\neq 0$. This is confirmed by the simulation response shown in Figure 3, where a slight improvement brought by the proposed observer in the estimation of $d(t)$ can be perceived.

5.4 Scenario 4

Let us now consider the delay from Scenario 2 $d(t)=0.4+0.2\sin(0.4t)$ with known lower bound for the delay $\underline{d}(t)=0.2$, and the input signal from Scenario 3 $u(t)=\sin(0.1t)$. The corresponding results are showed in Figure 4. It can be seen that the estimate obtained with $\underline{d}=0$ suffers much more from the loss of observability of the system (5) when $\dot{u}(t)=0$, which is the reason for the sudden changes in $\hat{d}(t)$ at approx. 16.5 s and 47.5 s.

5.5 Scenario 5

We will now analyze the behavior of the proposed observer under relatively big delay and known lower bound. Under such situation, the difference between the right-hand terms in the inequalities (3) and (9) becomes more significant, which should lead to a bigger difference between the performance of the observer without known lower bound ($\underline{d}=0$) and that of the proposed observer. More specifically, let us consider $d(t)$ as follows:

$$d(t)=\begin{cases}1.15 & \text{for } 0\leq t\leq 15\\ 1.6 & \text{for } 15<t\leq 30\\ 1.3 & \text{otherwise}\end{cases} \tag{45}$$

with $u(t)=\sin(0.1t)$, with a bound $\underline{d}=1$. The corresponding results are shown in Figure 5, where it can be seen that the proposed observer estimates $d(t)$ with a much reduced error and mitigates the impact of the loss of observability when $\dot{d}(t)=0$.

5.6 Scenario 6

Finally, Figure 6 shows the results obtained under the same conditions as Scenario 5, except for the delay being $d(t)=1.4+0.2\sin(0.4t)$. Also in this case, the benefit of

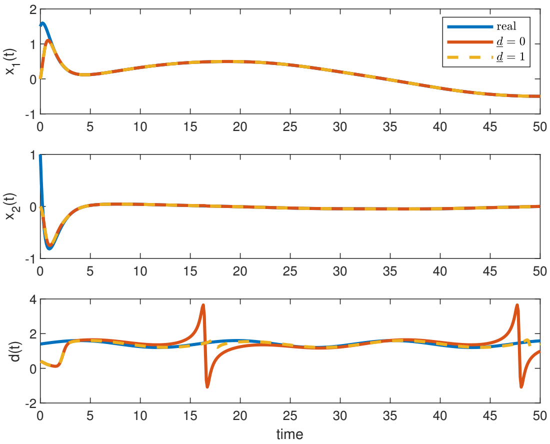

Figure 6. Simulation results (Scenario 6).

considering a lower bound for $\underline{d}$ is evident by looking at the delay estimation error.

6 Conclusions

This paper has proposed an improved observer design for joint state and delay estimation. In particular, the improvement with respect to a similar design previously proposed in the literature comes from the knowledge of a lower bound for the time-varying delay, which can be taken into account during the application of Taylor's theorem, so that a generally smaller remainder is obtained. The simulation results have shown that the proposed design does not improve the state estimation. On the other hand, the delay estimation is improved sensibly in cases where approximation errors become non-negligible or when the slowly changing nature of the input signal ($\dot{u} \approx 0$) causes loss of observability issues. In particular, it was observed that for large delays, the performance improvement is outstanding. As a side note, it has been observed so far that this type of method is fragile when the system is affected by parametric uncertainties or nonlinearities, so that future research should be devoted to increase the robustness properties of the observer.

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